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ON INNER DERIVATIONS OF MALCEV ALGEBRAS

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1. **Introduction.** This paper generalizes a result due originally to Sagle [4] on inner derivations. In 1955, A. I. Malcev introduced a new product defined by a commutator in an alternative algebra. He called this structure a Moufang-Lie algebra. Sagle [3] developed some of the structure theory of these algebras and named them Malcev algebras. A Malcev algebra A is defined to be a nonassociative algebra which satisfies the identities:

(i) $x^2 = 0$ for x in A,

(ii) $(xy)(xz) = ((xy)z)x + ((yz)x)x + ((zx)x)y$ for x, y, z in A.

Throughout this paper A will denote a finite-dimensional Malcev algebra over a field F of arbitrary characteristics unless otherwise specified. The product of any two elements x, y of A will be denoted by juxtaposition, xy. For x in A let $R(x)$ denote the linear map $a \to ax$ for every a in A and let $R(B)$ be the linear space spanned by all $R(y)$ for y in B. Let $J(A, A, A)$ be the linear space spanned by all elements of the form $J(x, y, z) = (xy)z + (yz)x + (zx)y$ for x, y, z in A. Recall that the J-nucleus N of A is defined by $N = \{x \in A : J(x, A, A) = 0\}$. Schafer [5] defines the Lie multiplication algebra $L(A)$ of an arbitrary nonassociative algebra. Let $[R(x), R(y)]$ be the commutator of any two elements $R(x)$, $R(y)$ where x and y are in A. Sagle [3] shows $L(A) = R(A) + [R(A), R(A)]$ if A is a Malcev algebra. A derivation of an algebra A is a linear map D of A such that $(xy)D = (xD)y + x(yD)$ for every x, y in A. A derivation D of a Malcev algebra is inner if D is in $L(A)$. The main result is: If A is a Malcev algebra over a field F of characteristic unequal to 2 or 3 and the Killing form on A and $L(A)$ is nondegenerate then every derivation of A is inner. From this result we obtain the fact that if F has zero characteristic, then every derivation of A, where A is a semisimple Malcev algebra, is an inner derivation.

2. **Inner derivations of Malcev algebras.** Recall that if A is a semisimple Malcev algebra, then A is a direct sum of ideals which are simple algebras.

LEMMA 1. *If A is a semisimple Malcev algebra over a field F of characteristic unequal to 2 or 3 and $f(x, y) =$* Tr $R(x)R(y)$, *for x, y in*

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