ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 2, Number 4, Fall 1972

# REAL REPRESENTATIONS OF SPLIT METACYCLIC GROUPS

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The number of real absolutely irreducible representations of a rather special class of metacyclic groups was determined in [2]. The results of [2] are extended here to the class of all split metacyclic groups. This class includes, for example, all groups having every Sylow subgroup cyclic (see [1, p. 112]).

Suppose $G$ is a split metacyclic group having cyclic subgroups $A = \langle a \rangle$ and $B = \langle b \rangle$, with $A \lhd G$, $AB = G$, and $A \cap B = 1$. Suppose $|A| = m$, $|B| = s$, and $b^{-1}ab = a^r$, with $0 < r < m$. Then $(m, r) = 1$ and $r^s \equiv 1 \pmod{m}$. We shall assume throughout that $r > 1$, for otherwise $G$ is abelian. Denote by $u$ the order of $r$ modulo $m$, i.e., $u$ is the least positive integer such that $r^u \equiv 1 \pmod{m}$. Then $u \mid s$.

If $\zeta \in \mathbf{C}$ is a primitive $m$th root of unity set $\varphi_i(a) = \zeta^i$. Then $\hat{A} = \{\varphi_0, \varphi_1, \cdots, \varphi_{m-1}\}$ is the set of all irreducible complex characters of $A$. It will be convenient to utilize a method due to Mackey (see [3]) for constructing all the irreducible characters of $G$. Mackey's construction is given in the context of locally compact groups, but it is not difficult to verify his results directly for finite groups.

Observe that $B$ acts as a permutation group on $\hat{A}$, via $\varphi_i{}^b(a) = \varphi_i(b^{-1}ab) = \varphi_i(a^r) = \zeta^{ir}$. For each $\varphi_i \in \hat{A}$ let us denote by $\mathcal{O}_i$ the $B$-orbit of $\varphi_i$ and by $B_i$ the stabilizer in $B$ of $\varphi_i$. Thus $|\mathcal{O}_i| = |B : B_i|$, the index of $B_i$ in $B$, for each $i$. If we set $u_i = |\mathcal{O}_i|$ it is easy to see that $u_i$ is the least positive integer such that $m \mid i(r^{u_i} - 1)$, and that $u_i \mid u$. Furthermore $B_i = \langle b^{u_i} \rangle$ and $|B_i| = s/u_i$.

If $\xi \in \mathbf{C}$ is a primitive $s$th root of unity then the irreducible characters of $B_i$ are given by $\{\psi_{ij} : 0 \leqq j \leqq s/u_i - 1\}$, where $\psi_{ij}(b^{u_i}) = \xi^{ju_i}$. Define characters $\chi_{ij}$ of $B_iA$ by setting

$$\chi_{ij}(b^{u_i\beta}a^\alpha) = \psi_{ij}(b^{u_i\beta})\varphi_i(a^\alpha) = \xi^{u_ij\beta}\zeta^{i\alpha}.$$

If one representative $\varphi_i$ is chosen from each orbit $\mathcal{O}_i$, and if the resulting characters $\chi_{ij}$ are induced up to characters of $G$, then the set $\{\chi_{ij}^G\}$ of induced characters is the full set of inequivalent irreducible complex characters of $G$.

Received by the editors June 12, 1970 and, in revised form, January 18, 1971.
*AMS* 1970 *subject classifications.* Primary 20C15, 20C30.