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APPLICATIONS OF THE THEORY OF IMAGINARY POWERS OF OPERATORS

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ABSTRACT. Imaginary powers of directional derivatives and of certain other operators are used to study semigroups which arise in the analysis of singular integral operators. Imaginary powers of directional derivatives are used to estimate the maximal functions and the Littlewood-Paley g-function of the Poisson integral on a Hilbert space.

I. **Introduction.** The purpose of this paper is to study some of the implications of the existence as bounded operators of purely imaginary powers of the infinitesimal generators of certain semigroups. The setting of the paper will be Classical Analysis on Hilbert Space.

Let H be a real separable Hilbert space and let $L_p(H)$ denote the Banach space of p-power integrable functions with respect to the normal distribution with variance parameter 1. Let $y \to T_y$ denote the regular representation of the additive group of H as isometries on $L_p(H)$. Fix p in $1 < p < \infty$. Let B denote a one-one Hilbert-Schmidt operator on H and let n_t denote the normal distribution on H with variance parameter $t/2$. Then $n_t \circ B^{-1}$ is a Borel probability measure on H; for f in $L_p(H)$, set

$$H_t(f) = \int_H T_y f \, dn_t \circ B^{-1}(y),$$

$$P_z(f) = \int_0^\infty H_t(f) N_t(z) \, dt/t$$

where $N_t(z) = (\pi t)^{-1/2} z \exp(-t^{-1}z^2)$. $P_z(f)$ is the Poisson integral of f. If $(-D_h)$ denotes the infinitesimal generator of the translation semigroup T_{th}, $t > 0$, and if $(-T)$ denotes the infinitesimal generator of P_z, $z > 0$, then $(D_h)^{ic}$ and T^{ic} are strongly continuous groups of

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