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GAUSSIAN MEASURE IN HILBERT SPACE AND APPLICATIONS IN NUMERICAL ANALYSIS

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ABSTRACT. The numerical analyst is often called upon to estimate a function from a very limited knowledge of its properties (e.g. a finite number of ordinate values). This problem may be made well posed in a variety of ways, but an attractive approach is to regard the required function as a member of a linear space on which a probability measure is constructed, and then use established techniques of probability theory and statistics in order to infer properties of the function from the given information. This formulation agrees with established theory, for the problem of optimal linear approximation (using a Gaussian probability distribution), and also permits the estimation of nonlinear functionals, as well as extension to the case of "noisy" data.

1. **Introduction.** The problem which is central to the subject to be discussed occurs frequently in numerical analysis and the interpretation of experimental data. Typically, we may be given the ordinate values of a function, measured at a finite number of abscissae, and wish to interpolate, i.e. make a reasonable estimate of the function value at some other abscissa. This problem may be regarded in two parts:

(i) Construct an estimator for the required unknown value.

(ii) Determine how accurate the estimated value is likely to be.

The approach traditionally taken by numerical analysts has been to assume an algebraic form for the function in question, e.g., a polynomial of specified order, to determine the assignable parameters (coefficients) by forcing the function to satisfy the given constraints, and subsequently to refer to this constructed function for estimating any other required information.

A statistician, on the other hand, might assume a joint probability distribution, e.g., multivariate normal, for the known and unknown quantities, and thence determine a conditional distribution for the required values.

Each of these approaches has its own advantages and shortcomings. We shall be working towards a generalisation which retains some of

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