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INTRODUCTION TO GENERAL THEORY OF REPRODUCING KERNELS

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1. **Introduction.** The theory of reproducing kernels is of fairly recent origin. The beginnings go back to the work of G. Szegö (1921) and S. Bergman (1922). We shall give formal definitions later but at this stage a rough description may be helpful.

Consider a class F of functions $P \to f(P)$ defined on some set S. A function of two arguments $K(P, Q)$ is a reproducing kernel for the class F if for each $f \in F$ we have

$$f(P) = \int f(Q)K(P, Q)\, dQ \tag{1.1}$$

where the integral is taken over S or over some proper subset of S.

This formulation is a little too general for our purposes, but it gives the idea. With this formulation it is easy to give examples of reproducing kernels some of which are of quite an old vintage. Cauchy's integral is such a case. Here F is the class of all functions holomorphic in a domain D bounded by a simple closed rectifiable oriented curve C. We assume every $f \in F$ to be continuous in $C \cup D$ and can then write for z in D

$$f(z) = \frac{1}{2\pi i} \int_C \frac{f(t)}{t - z}\, dt. \tag{1.2}$$

This is of the form (1.1) if we take

$$K(z, t) = \frac{1}{2\pi i} \frac{1}{t - z}. \tag{1.3}$$

There is of course no need to assume that D is simply-connected; if not, C will have to be the total boundary.

Various generalizations of Cauchy's integral are known. Thus we can let f be matrix-valued, P an n by n matrix, and $f(P)$ be given by a resolvent integral. This would already be a deviation from the pattern set by (1.1).

Another formula which comes to mind is the following:

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