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SOME GENERALIZATIONS OF MEHLER'S FORMULA[1]

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ABSTRACT. A number of earlier results of the authors, involving the classical Hermite polynomials, are applied to prove two generalizations of some interesting extensions of the well-known Mehler formula, given recently by Carlitz.

1. **Introduction.** Let $H_n(z)$ denote the Hermite polynomial defined by

$$\sum_{n=0}^{\infty} H_n(z) \frac{t^n}{n!} = \exp(2zt - t^2). \tag{1.1}$$

In an attempt to unify several extensions of the well-known Mehler formula [4, p. 198]

$$\begin{aligned}\sum_{n=0}^{\infty} H_n(x)H_n(y) \frac{t^n}{n!} \\ = (1 - 4t^2)^{-1/2} \exp\left\{\frac{4xyt - 4(x^2 + y^2)t^2}{1 - 4t^2}\right\},\end{aligned} \tag{1.2}$$

given recently by Carlitz [2], we proved the following general formulas [5]:

$$\begin{aligned}\sum_{m,n,p=0}^{\infty} & H_{n+p+r}(x)H_{p+m+s}(y)H_{m+n}(z) \frac{u^m}{m!}\frac{v^n}{n!}\frac{w^p}{p!} \\ &= S_1 \sum_{k=0}^{\min(r,s)} 2^{2k}k! \binom{r}{k}\binom{s}{k}\left(\frac{w - 2uv}{\sqrt{\{(1-4u^2)(1-4v^2)\}}}\right)^k \\ &\cdot H_{r-k}\left(\frac{(x - 2vz)(1 - 4u^2) - 2(y - 2uz)(w - 2uv)}{\sqrt{\{\Delta(1 - 4u^2)\}}}\right) \\ &\cdot H_{s-k}\left(\frac{(y - 2uz)(1 - 4v^2) - 2(x - 2vz)(w - 2uv)}{\sqrt{\{\Delta(1 - 4v^2)\}}}\right),\end{aligned} \tag{1.3}$$

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