ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 2, Number 2, Spring 1972

DECOMPOSITIONS OF DIRECT SUMS OF CYCLIC p-GROUPS

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Throughout we will be considering only p-primary Abelian groups G which are direct sums of cyclic groups. Such groups have many basic subgroups. Recall that B is a basic subgroup of G if B is a direct sum of cyclic groups (which is automatic here), pure, and G/B is divisible. Since G is a direct sum of cyclic groups, G itself is basic in G. If $G = \sum_{i=1}^{\infty} Zx_i$, where $o(x_i) = p^{n_i}$, $n_1 < n_2 < \cdots$, then $B = \sum_{i=1}^{\infty} Z(x_i - p^{n_{i+1}-n_i}x_{i+1})$ is basic in G with $G/B \approx Z(p^{\infty})$[**1**, Lemma 31.1, p. 103]. Let G be any direct sum of cyclics, and suppose B is basic in G with $G/B \approx Z(p^{\infty})$. Write $G = \sum_{i=1}^{\infty} G_i$, where G_i is a direct sum of cyclic groups of order p^{n_i}, $n_1 < n_2 < \cdots$, and where no $G_i = 0$. Tarwater [**6**] showed that $G = X_1 \oplus X$ with

$$B = (B \cap X_1) \oplus X, \qquad X_1 = \sum_{i=1}^{\infty} Zx_i,$$

$$B \cap X_1 = \sum_{i=1}^{\infty} Z(x_i - p^{n_{i+1}-n_i}x_{i+1}),$$

and $o(x_i) = p^{n_i}$. In particular, if C is any basic subgroup of G with $G/C \approx Z(p^{\infty})$, then there is an automorphism α of G such that $\alpha(B) = C$. More generally, he indicated in [**5**] that if G is any direct sum of cyclic groups with basic subgroups B and C such that G/B and G/C are isomorphic and have (the same) finite rank, then there is an automorphism α of G such that $\alpha(B) = C$. The idea is to show that $G = X_1 \oplus \cdots \oplus X_n = Y_1 \oplus \cdots \oplus Y_n$ with $B = (B \cap X_1) \oplus \cdots \oplus (B \cap X_n)$, $C = (C \cap Y_1) \oplus \cdots \oplus (C \cap Y_n)$, $X_i \approx Y_i$, and with $X_i/(B \cap X_i) \approx Y_i/(C \cap Y_i) \approx Z(p^{\infty})$. Now P. Hill [3] has proved that if G is any direct sum of cyclic groups and B and C are basic in G with $G/B \approx G/C$, then there is an automorphism α of G such that $\alpha(B) = C$. Hill's proof involves extending height-preserving automorphisms of subgroups, and employs a two stage transfinite induction. One would hope that the general case follows from the case when $G/B \approx G/C \approx Z(p^{\infty})$, or at least that most of the group

Received by the editors November 19, 1970 and, in revised form, December 10, 1970.

AMS 1970 *subject classifications.* Primary 20K10, 20K30; Secondary 20K25.