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ON THE WEIERSTRASS PREPARATION THEOREM

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Introduction. Suppose that R is a commutative ring with identity, X is an indeterminate over R, and $S = R[[X]]$ is the formal power series ring. In [1, §3, Proposition 6], the following result (Weierstrass Preparation Theorem) is proved when R is a local ring, complete in its maximal ideal adic topology: Suppose that $f = \sum_{i=0} a_i X^i \in S$, where, for some $n \geqq 1$, a_n is a unit of R and $(a_0, a_1, \cdots, a_{n-1}) \subseteq M$, the maximal ideal of R. Then there exists a unique pair $u, F \in S$ such that u is a unit of S and F is a monic polynomial of degree n with the property that the coefficients of X^i in F, for $i < n$, are elements of M, and such that $f = uF$.

In this paper we extend this result, together with Proposition 5 of [1, §3] and its Corollary, to the case when R is any commutative ring with identity and $f = \sum_{i=0} a_i X^i$ satisfies the property that, for some $n \geqq 1$, a_n is a unit of R, while the ideal $A = (a_0, a_1, \cdots, a_{n-1})$ generates a complete Hausdorff topology on R.

In §1 we give the notation and terminology used throughout the paper, and we prove three results needed in §2. §2 contains our main results.

All rings considered in this paper are assumed to be commutative and to contain an identity element. The symbols ω and ω_0 are used throughout the paper to denote the sets of positive and nonnegative integers, respectively. A collection of ideals $\{A_k\}_{k \in \omega}$ of the ring R will be called a *d-sequence* provided that for any $n, m \in \omega$ there exists a $u \in \omega$, depending on n and m, such that $A_u \subseteq A_n \cap A_m$.

1. **Preliminaries.** Let R be a ring, and let Ω be the topology induced on R by the d-sequence $\{A_k\}_{k \in \omega}$ of R. We write (R, Ω) to denote the topological ring R under the topology Ω. It is well known that (R, Ω) is Hausdorff if and only if $\bigcap_{k \in \omega} A_k = (0)$. We say that (R, Ω) is *complete* if each Cauchy sequence of R converges to a point of R. If there exists an ideal M of R such that $M^k = A_k$ for each $k \in \omega$, then the topology Ω is called the M-adic topology, and we write (R, M) instead of (R, Ω) in this case.

(1.1) LEMMA. *Let R be a ring and suppose that the topology Ω induced on R by the d-sequence $\{A_k\}_{k \in \omega}$ of R is Hausdorff. Then,*

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