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THE AUTOMORPHISM GROUP OF AN EXTRASPECIAL p-GROUP

DAVID L. WINTER

1. Let p be a prime. The finite p-group P is called special if either (i) P is elementary abelian or (ii) the center, commutator subgroup and Frattini subgroup of P all coincide and are elementary abelian. A nonabelian special p-group whose center has order p is called an extraspecial p-group. It is possible to give a uniform treatment of the subject of automorphisms for all the possible isomorphism types of extraspecial p-groups and so some cases that are more or less known are included here. The result when p is odd and P has exponent p^2 leads to an interesting subgroup of the symplectic group $\mathrm{Sp}(2n, q)$, q a power of p, $n > 1$. This subgroup is the semidirect product of $\mathrm{Sp}(2n-2, q)$ and a normal special p-group of order q^{2n-1} whose center has order q.

THEOREM 1. *Let p be a prime and let P be an extraspecial p-group of order p^{2n+1}. Let I be the group of inner automorphisms and let H be the normal subgroup of* $\mathrm{Aut}\, P$ *consisting of all elements of* $\mathrm{Aut}\, P$ *which act trivially on* $Z(P)$. *Then* $\mathrm{Aut}\, P = \langle \theta \rangle H$ *where θ has order $p-1$, $H \cap \langle \theta \rangle = \langle 1 \rangle$ and H/I is isomorphic to a subgroup of* $\mathrm{Sp}(2n, p)$. *Furthermore,*

(a) *If p is odd and P has exponent p, $H/I \cong \mathrm{Sp}(2n, p)$ of order $p^{n^2}\prod_{i=1}^{n}(p^{2i}-1)$.*

(b) *If p is odd and P has exponent p^2, H/I is the semidirect product of* $\mathrm{Sp}(2n-2, p)$ *and a normal extraspecial group of order p^{2n-1}. (If $n = 1$, H/I has order p.)*

(c) *If $p = 2$, H/I is isomorphic to the orthogonal group $O_\epsilon(2n, 2)$ of order $2^{n(n-1)+1}(2^n - \epsilon)\prod_{i=1}^{n-1}(2^{2i}-1)$. Here $\epsilon = 1$ if P is isomorphic to the central product of n dihedral groups of order 8 and $\epsilon = -1$ if P is isomorphic to the central product of $n-1$ dihedral groups of order 8 and a quaternion group.*

COROLLARY 1. *Let p be an odd prime and let P be an extraspecial p-group of exponent p^2. There is a nonidentity element of $P/Z(P)$ left fixed by every automorphism of P.*

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