THE AUTOMORPHISM GROUP OF AN EXTRASPECIAL $p$-GROUP

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1. Let $p$ be a prime. The finite $p$-group $P$ is called special if either (i) $P$ is elementary abelian or (ii) the center, commutator subgroup and Frattini subgroup of $P$ all coincide and are elementary abelian. A nonabelian special $p$-group whose center has order $p$ is called an extraspecial $p$-group. It is possible to give a uniform treatment of the subject of automorphisms for all the possible isomorphism types of extraspecial $p$-groups and so some cases that are more or less known are included here. The result when $p$ is odd and $P$ has exponent $p^2$ leads to an interesting subgroup of the symplectic group $Sp\left(2n, q\right)$, $q$ a power of $p$, $n > 1$. This subgroup is the semidirect product of $Sp\left(2n - 2, q\right)$ and a normal special $p$-group of order $q^{2n-1}$ whose center has order $q$.

**Theorem 1.** Let $p$ be a prime and let $P$ be an extraspecial $p$-group of order $p^{2n+1}$. Let $I$ be the group of inner automorphisms and let $H$ be the normal subgroup of $Aut\ P$ consisting of all elements of $Aut\ P$ which act trivially on $Z(P)$. Then $Aut\ P = \langle \theta \rangle H$ where $\theta$ has order $p - 1$, $H \cap \langle \theta \rangle = \langle 1 \rangle$ and $H/I$ is isomorphic to a subgroup of $Sp\left(2n, p\right)$. Furthermore,

(a) If $p$ is odd and $P$ has exponent $p$, $H/I \cong Sp\left(2n, p\right)$ of order $p^n \prod_{i=1}^{n}(p^{2i} - 1)$.

(b) If $p$ is odd and $P$ has exponent $p^2$, $H/I$ is the semidirect product of $Sp\left(2n - 2, p\right)$ and a normal extraspecial group of order $p^{2n-1}$. (If $n = 1$, $H/I$ has order $p$.)

(c) If $p = 2$, $H/I$ is isomorphic to the orthogonal group $O_e(2n, 2)$ of order $2^{n(n-1)+1}(2^n - \epsilon)\prod_{i=1}^{n-1}(2^{2i} - 1)$. Here $\epsilon = 1$ if $P$ is isomorphic to the central product of $n$ dihedral groups of order 8 and $\epsilon = -1$ if $P$ is isomorphic to the central product of $n - 1$ dihedral groups of order 8 and a quaternion group.

**Corollary 1.** Let $p$ be an odd prime and let $P$ be an extraspecial $p$-group of exponent $p^2$. There is a nonidentity element of $P/Z(P)$ left fixed by every automorphism of $P$.

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