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THE IDEAL TRANSFORM IN A GENERALIZED KRULL DOMAIN

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Introduction. Let R be a commutative integral domain with identity and let L denote the quotient field of R. R is called a generalized Krull domain [5] if there is a family F of rank one valuation on L satisfying the following, which we label as (E).

(E)	(1)	Each $v \in F$ has rank one.
	(2)	$R = \bigcap\{R_v \mid v \in F\}$.
	(3)	$R_v = R_{P(v)},\ v \in F$.
	(4)	F is of finite character [5].

In this case, F is called the family of essential valuations of R.

In [6], Nagata defined the transform $T(A)$ of an ideal A of R as follows: $T(A) = \bigcup_{n=1}^{\infty} R : A^n$, where $R : B = \{x \in L \mid xB \subseteq R\}$ for any ideal B of R. Nagata [6] characterized the transform of an ideal A when R is a Krull domain and showed that when R is Krull, the transform of any ideal of R is the transform of a finitely generated ideal. Brewer in [1] characterized the transform of any finitely generated ideal when R is an arbitrary integral domain.

In §1 of this paper, we obtain a characterization of the transform of an arbitrary ideal of R, when R is a generalized Krull domain, that generalizes Nagata's and Brewer's results. This characterization provides the basis for a "transform algebra." §2 contains two examples to show how the results of §1 fail when the finite character assumption on F is dropped.

1. Let R be a generalized Krull domain with quotient field L and family F of essential valuations. For any nonzero ideal A of R and any $v \in F$, put $v(A) = \inf\{v(a) \mid a \in A\}$. Since F is of finite character, it follows that $v(A) \neq 0$ for only finitely many $v \in F$. We let $F_A = \{v \in F \mid v(A) \neq 0\}$.

Theorem 1.1. *Let A be any nonzero ideal of R. Then* $T(A) = \bigcap\{R_w \mid w(A) = 0\}$.

Proof. Let $x \in T(A)$. Then $xA^n \subseteq R$ for some n. So for $w \in F - F_A$

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