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$\lambda(n)$-CONVEX FUNCTIONS

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ABSTRACT. $\lambda(n)$-convex functions include as special cases the classical convex and generalized convex functions. Relationships between convexity and $\lambda(n)$-convexity are noted for certain types of $\lambda(n)$-convexity, and some relationships among various types of $\lambda(n)$-convexity are derived. In addition it is shown that for a disconjugate linear homogeneous differential equation solutions to the corresponding differential inequality are $\lambda(n)$-convex for all values of the "ordered partition" $\lambda(n)$ of the positive integer n.

Introduction. A real valued function s is said to be convex with respect to an "n-parameter family" F of functions on an interval I of the real numbers if whenever there is an $f \in F$ such that $s - f$ has n zeros on I, then $s - f$ is nonpositive on the interval between the last two of the zeros and changes sign only at each of the other zeros except the first. In this paper we consider convexity in the case that $s - f$ has n zeros *counting multiplicity*, give a definition for this new type of convexity and, under appropriate assumptions on F, establish some relationships among various types of convex functions where $s - f$ has at least one double zero.

Functions convex with respect to n-parameter families of functions are often called *generalized convex functions*. An extensive bibliography on this subject is found in the interesting recent article [2] by J. H. B. Kemperman.

All functions considered are real valued, $f^{(j)}(x)$ denotes the jth derivative of f at x and I^0 denotes the interior of the interval I of the real numbers. In the subsequent discussion we assume that $n \geqq 3$. F denotes an n-parameter family unless specified otherwise.

We also remark here that there is an obvious analogous idea of "concave functions" obtained by replacing "$>$" by "$<$" in (1) and (5). It is clear that each result for convex functions has an analog for concave functions, and where convenient we shall use these results for concavity.

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