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BOUNDARY VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS[1]

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1. Consider the nth order differential equation

$$y^{(n)} = f(t, y, y', \cdots, y^{(n-1)}) \qquad (n \geqq 2) \tag{1.1}$$

where $f(t, x_0, x_1, \cdots, x_{n-1})$ and $f_i(t, x_0, \cdots, x_{n-1})$ are continuous on $I \times R^n$ where I is an open subinterval of R and where

$$f_i(t, x_0, \cdots, x_{n-1}) = \frac{\partial f}{\partial x_i}(t, x_0, \cdots, x_{n-1}), \quad i = 0, 1, \cdots, n-1.$$

After some preliminaries in §1, we devote §2 to establishing necessary and (or) sufficient conditions in order that there exist at most one solution of (1.1) satisfying the boundary conditions

$$y^{(i)}(c) = \alpha_i, \qquad y^{(k)}(d) = \beta, \tag{1.2}$$

or

$$y^{(k)}(c) = \alpha, \qquad y^{(i)}(d) = \beta_i, \tag{1.3}$$

when $c, d \in I$, $c < d$, $i = 0, 1, 2, \cdots, n-2$, $0 \leqq k \leqq n-1$ is fixed, and the α_i and β_i are constants. We shall refer to (1.2) as an $(n; k)$ BVP for equation (1.1) and (1.3) as a $(k; n)$ BVP. (We use n rather than $n-1$ to avoid confusion when $k = n-1$.) Our technique, similar to that used in [1] for $n = 2$, involves studying the behaviour of solutions of the variational equation

$$z^{(n)} = \sum_{i=0}^{n-1} f_i(t, y(t), \cdots, y^{(n-1)}(t))z^{(i)} \tag{1.4}$$

where $y(t)$ is a solution of (1.1). For $n = 2$ and 3 it has been shown [2], [3] that if solutions to the n-point BVP,

$$y(t_i) = \alpha_i, \qquad t_i < t_{i+1}, \qquad 1 \leqq i \leqq n-1, \tag{1.5}$$

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