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# A NOTE ON THE INTERSECTION OF THE POWERS OF THE JACOBSON RADICAL

MAX D. LARSEN AND AHMAD MIRBAGHERI

1. **Introduction and preliminaries.** All rings will be assumed to have identity. If $R$ is a ring, $J = J(R)$ will denote its Jacobson radical. The purpose of this note is to establish conditions on $R$ such that $\bigcap_{i=1}^{\infty} J^i = 0$. In particular, we show that if $R$ is a right Noetherian $J$-prime ring such that every ideal of $R$ is a principal right ideal, and in addition, $J$ is a principal left ideal, then $J$ is the nilpotent radical of $R$ or $\bigcap_{i=1}^{\infty} J^i = 0$. Further, we show that $\bigcap_{i=1}^{\infty} J^i = 0$ if $R$ is a right Noetherian ring, $J$ is a principal right ideal, and $\bigcap_{i=1}^{\infty} J^i$ is a finitely generated left ideal of $R$. The methods of J. C. Robson [5] are used throughout, and Theorems 3.5 and 5.3 of Robson's paper are generalized.

A ring is called an *ipri-ring* (*ipli-ring*) if every ideal is a principal right (left) ideal [5, p. 127]. Condition ($\alpha$) is said to hold in $R$ if $ab$ being regular in $R$ is equivalent to both $a$ and $b$ being regular in $R$. Combining [1, Theorems 4.1 and 4.4, pp. 212-213] and [4, Corollary 2.6, p. 603] one sees that if $R$ is a semiprime right Noetherian ring, then ($\alpha$) holds in $R$. A ring $R$ is said to be *$J$-prime* (*$J$-simple*) if $R/J$ is a prime (simple) ring. The nilpotent radical of a ring is denoted by $W$ and *$W$-simple* is defined similarly. The symbol $\subset$ will denote proper containment.

A result important to our work is the following lemma [3, p. 200]:

LEMMA 1.1. *For any ring $R$, if $G$ is a nonzero ideal of $R$ finitely generated as a right (left) ideal of $R$ and $G \subseteq J = J(R)$, then $GJ \subset G$ ($JG \subset G$).*

LEMMA 1.2. *Let $R$ be a right Noetherian $J$-prime ipri-ring. If $T$ is an ideal of $R$ such that $T \not\subseteq J$, then $J \subset T$.*

PROOF. Let $B = T + J = bR$ and $J = aR$. Assume $J \subset B$. Then the image of $B$ in $R/J$ is a nonzero ideal and hence the image of $b$ is regular since $R/J$ is a prime right Noetherian ring [5]. Since $J \subset bR$, we have $J = bJ$. Hence $J \subset T + J^2$ and there exist $t \in T$ and $r \in R$ such that $a(1 - ar) = t$. But $1 - ar$ is a unit in $R$ so $a \in T$. Thus $J \subset T$.

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