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FIXED LENGTH CONFIDENCE INTERVALS FOR PARAMETERS OF THE NORMAL DISTRIBUTION BASED ON TWO-STAGE SAMPLING PROCEDURES[1]

L.H. KOOPMANS AND CLIFFORD QUALLS

1. **Introduction and summary.** In many industrial situations the statistician is required to estimate a statistical parameter not only with prescribed confidence or reliability but also with prescribed precision. The most natural procedure is to construct a confidence interval for the parameter for which both the confidence coefficient, $1 - \alpha$, and the length of the interval, $2L$, can be specified in advance.

In this paper fixed length confidence intervals based on two-stage sampling procedures are proposed for the variance and coefficient of variation in the case of a single normal distribution and for the difference in means and ratio and difference of variances in the case of two populations.

The usual one-stage sampling methods do not lead to confidence intervals with both prescribed confidence coefficient and length for any of the parameters we consider. In fact, no one-stage confidence interval can be constructed for any of these parameters which satisfy both requirements. (See, e.g. [1], [2].) The reason for this difficulty can be seen, heuristically, by studying the classical confidence interval for the mean μ of a normal distribution when the variance σ^2 is also unknown. The endpoints are $\bar{X} \pm t_\alpha s/\sqrt{n}$ where $\bar{X}$ is the mean of a sample of size n, t_α is a percentile of the Student's t distribution, and s^2 is the unbiased sample variance. Now our ignorance of the magnitude of σ and consequently of its estimate s makes it impossible to select, in advance, a sample size n which will guarantee a prescribed bound on the length of this confidence interval.

In a pioneering paper [7] Stein showed how to overcome this problem by employing two stages of sampling. The first sampling stage is used to obtain an estimate of σ. If the usual $100(1 - \alpha)\%$ confidence interval (above) computed for the first sample is not short enough to meet the length requirement, a second sample size based on the esti-

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