ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 1, Number 3, Summer 1971

BEST CHEBYSHEV QUADRATURES

RICHARD FRANKE[1]

Introduction. The Chebyshev quadrature

$$(1) \qquad \int_{-1}^{1} x(s)ds \cong u \sum_{k=1}^{m} x(a_{k-1}),$$

will be considered. Following the method of Sard [1], it will be assumed that $d^{n-1}x/ds^{n-1}$ is absolutely continuous, and that the approximation is precise for degree $\leqq n-1$. Under these assumptions, the remainder, or error term,

$$(2) \qquad Rx = \int_{-1}^{1} x(s)dx - u \sum_{k=1}^{m} x(a_{k-1})$$

may be written in the form (see [1, p. 25])

$$(3) \qquad Rx = \int_{-1}^{1} K(t) \frac{d^n x}{ds^n}(t)dt.$$

One possible appraisal of the magnitude of the error term is obtained by applying the Schwarz inequality to (3), obtaining

$$|Rx|^2 \leqq J \int_{-1}^{1} \left[\frac{d^n x}{ds^n}(t) \right]^2 dt,$$

where

$$(4) \qquad J = \int_{-1}^{1} [K(t)]^2 dt.$$

Any L_p norm ($p \geqq 1$) of $K(t)$ could be considered. The L_2 norm was chosen because of the resulting simplicity of the calculations.

This paper will obtain "best" Chebyshev quadratures in the sense of Sard, i.e., those which minimize J. We will require that $-1 \leqq a_0 < a_1 < \cdots < a_{m-1} \leqq 1$, and that the a_k be symmetric, i.e., $a_{i-1} = -a_{m-i}$, $i = 1, \cdots, m$. Precision zero will be required in all cases, thus $n \geqq 1$ and $u = 2/m$.

Received by the editors November 17, 1969 and, in revised form, January 29, 1970.

AMS 1970 subject classifications. Primary 41A55, 65D30.

[1]Present address: Naval Postgraduate School, Monterey, California 93940.