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COUNTABLY RECOGNIZABLE CLASSES OF GROUPS[1]

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I. **Introduction.** A class Σ of groups is a collection of groups containing the unit group E and closed under the taking of isomorphisms. Let Σ be a class of groups:

(i) $s(\Sigma)$ is the class of all groups which are subgroups of Σ groups.

(ii) $q(\Sigma)$ is the class of all groups which are quotients of Σ groups.

(iii) $L(\Sigma)$ is the class of all groups in which every finitely generated subgroup is a Σ group.

If $L(\Sigma) \subset \Sigma$, Σ is said to satisfy the local theorem. If Σ satisfies the local theorem and $s(\Sigma) = \Sigma$, then the class Σ is determined in a certain sense by the finitely generated groups in Σ.

In this paper, we are interested in classes of groups determined by their countable subgroups. In the sequel, the word countable will mean countably infinite or finite.

DEFINITION 1.1. Let Σ be a class of groups. $C(\Sigma)$ is the class of all groups G such that every countable subgroup of G is a Σ group.

DEFINITION 1.2. A class of groups Σ is countably recognizable if $C(\Sigma) \subset \Sigma$.

Observe that if Σ satisfies the local theorem, then Σ is countably recognizable. Further, if $s(\Sigma) = \Sigma$, then Σ is countably recognizable if and only if $C(\Sigma) = \Sigma$.

The notion of a countably recognizable class of groups is due to R. Baer [**1**]. In the paper [**1**], it is shown that many classes of groups which do not satisfy the local theorem are countably recognizable. There are other isolated theorems of this type in the literature: e.g., [**6**, p. 219] shows that the class of ZA groups is countably recognizable: see also [**10**, p. 349] for a theorem of this type.

In this paper, we add several classes to the list of countably recognizable classes. Let Σ be countably recognizable and assume $s(\Sigma) = \Sigma$. Then the following classes are also countably recognizable:

(1) The class of groups G such that every simple factor G is a Σ group (Theorem 4.2).

(2) The class of groups G such that every principal factor of every subgroup of G is a Σ group (Theorem 5.2).

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