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GENERALIZED POLAR COORDINATE TRANSFORMATIONS FOR DIFFERENTIAL SYSTEMS [1]

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1. **Introduction.** If $m(t)$ and $k(t)$ are real-valued, continuous functions on an interval I on the real line, and $m(t) > 0$ for $t \in I$, then it is well known that under the polar coordinate transformation

$$u(t) = \rho(t) \sin \theta(t), \qquad m(t)u'(t) = \rho(t) \cos \theta(t), \tag{1.1}$$

the differential equation

$$\mathfrak{L}[u](t) \equiv [m(t)u'(t)]' - k(t)u(t) = 0, \qquad t \in I, \tag{1.2}$$

is equivalent to the nonlinear differential system

$$\begin{aligned} &\text{(a)} \quad \theta'(t) = q(t; \sin \theta(t), \cos \theta(t)), \quad \text{where} \\ &\qquad q(t; s,c) = \frac{c^2}{m(t)} - k(t)s^2, \\ &\text{(b)} \quad \rho'(t) = \left\{ \left[\frac{1}{m(t)} + k(t) \right] \sin \theta(t) \cos \theta(t) \right\} \rho(t). \end{aligned} \tag{1.3}$$

To the present author it appears impossible to ascribe the introduction of the transformation (1.1) to any specific person, for the use of polar coordinates in the study of differential systems is of long standing, appearing in particular in the perturbation theory of two-dimensional real autonomous dynamical systems. The first published use of this substitution in the derivation of certain results of the Sturmian theory for a linear homogeneous differential equation (1.2) appears to be that of Prüfer [11], however, and in the literature this substitution is widely known as the Prüfer transformation of (1.2).

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