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DECISIVE CONVERGENCE SPACES, FRÉCHET SPACES AND SEQUENTIAL SPACES

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Decisive convergence spaces were introduced in [9], where it is shown that they provide a solution to Question 1 of [3]. In this paper we show that the S-spaces of Čech [4] and the Fréchet spaces of Arhangel'skiĭ [1] can be identified either the pretopological modifications of decisive convergence spaces or the pretopological quotients of decisive topologies, and that the sequential spaces of Franklin [6] can be characterized as either the topological modifications of decisive convergence spaces or the topological quotients of decisive topologies. The well-known characterizations of Fréchet and sequential spaces as certain quotients of metrizable spaces are obtained (in a somewhat generalized form) as a by-product. Indeed, using the convergence space approach to quotient maps developed in [8], we are able to display a natural transition from first-countable spaces to almost first-countable spaces to Fréchet spaces to sequential spaces as increasingly more general quotients of pseudo-metric spaces.

1. Our notation and terminology agree with that of [3], [8], and [9], and the reader is asked to refer to one of these sources for the basic information about convergence spaces. Starting with a convergence space (S, q), we shall refer to $\pi(q)$, the finest pretopology on S coarser than q, as the *pretopological modification* of q and to $\lambda(q)$, the finest topology on S coarser than q, as the *topological modification* of q. As in [9], we use the abbreviation "npuf" for "nonprincipal ultrafilter". Given a convergence space (S, q), let $\mathcal{G}_q(x)$ be the filter obtained by intersecting all nonprincipal ultrafilters which fail to q-converge to x; (S, q) is said to be *decisive* if a npuf $\mathcal{F}$ fails to q-converge to x whenever $\mathcal{F} \geqq \mathcal{G}_q(x)$. Another characterization of decisive convergence spaces can be stated as follows: a subset B of S is said to be *q-decisive for x* if every npuf $\mathcal{F}$ which contains B q-converges to x; then (S, q) is decisive if and only if, for each x in S, every npuf which q-converges to x contains a q-decisive set for x (Theorem 1, [9]). A convergence space (S, q) is *first-countable* (or, more formally, satisfies the first axiom of countability) if each filter which q-converges to x contains a filter with a countable filter base which q-converges to x. The real line with its usual topology is

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