ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 1, Number 2, Spring 1971

A NECESSARY AND SUFFICIENT CONDITION FOR THE OSCILLATION OF SOME LINEAR SECOND ORDER DIFFERENTIAL EQUATIONS

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1. **Introduction.** The ordinary differential equation

$$y'' + p(t)y = 0 \tag{1.1}$$

is called oscillatory if solutions have an infinite number of zeros in $[a, \infty)$ and disconjugate if no solution has more than one zero in $[a, \infty)$. The problem of oscillation for (1.1) has a long history, which can be surveyed by referring to [2]. In [1] we proved that if the integral of p is not "too negative"—a notion which we make precise in §3—then (1.1) is disconjugate, if and only if, the integral inequality

$$\nu(t) \geqq P(t) + \int_t^\infty \nu^2(s)ds \tag{1.2}$$

has a solution $\nu \in C(a, \infty)$. Here, P is an averaged integral of p, which means

$$P(t) = \int_t^\infty p(s)ds \tag{1.3}$$

if the integral in (1.3) exists as an improper integral. We also showed in [1] that (1.1) is disconjugate, if and only if, an integral inequality of the form

$$\nu(t) \geqq Q(t) + \int_t^\infty E(t, s)\nu^2(s)ds \tag{1.4}$$

has a solution $\nu \in C(a, \infty)$. Here, Q and E are nonnegative functions depending upon P.

In §2 of this note we give a necessary and sufficient condition in terms of nonnegative P and Q for the existence of a solution to (1.4). In §3 we apply this result to the oscillation problem for (1.1) to obtain a necessary and sufficient condition for the disconjugacy of (1.1). Since (1.2) is of the same general form as (1.4), we also formulate a necessary and sufficient condition for the disconjugacy of (1.1), by using (1.2), provided $P \geqq 0$.

Received by the editors September 22, 1969 and, in revised form, October 21, 1969.

AMS 1970 *subject classifications.* Primary 34C10.