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EIGENFUNCTION EXPANSIONS AND SCATTERING THEORY FOR PERTURBATIONS OF $-\Delta$

NORMAN SHENK AND DALE THOE

1. **Survey of results.** Let Ω be the unbounded domain exterior to a compact C^2 hypersurface Γ in R^n ($n \geqq 2$) and $q(x)$ a real-valued function such that $e^{2a|x|}q(x)$ is bounded and uniformly α-Hölder continuous in $\Omega \cup \Gamma$ for certain constants $a > 0$ and $0 < \alpha < 1$.

We let H denote the selfadjoint operator $-\Delta + q$ in $L^2(\Omega)$ acting on functions which are zero on Γ. Specifically, we define

$$(1.1)\qquad \begin{aligned} D(H) &= \{g : (\partial/\partial x)^\alpha g \in L^2(\Omega) \text{ for } |\alpha| \leqq 2 \text{ and } g|_\Gamma = 0\}, \\ Hg &= -\Delta g + qg \text{ for } g \in D(H). \end{aligned}$$

Here differentiation is interpreted in the space $\mathfrak{D}'(\Omega)$ of distributions on Ω and $g|_\Gamma$ is interpreted in an L^2 sense (see §4).

We treat H as a perturbation of the selfadjoint operator $H_0 = -\Delta$ in $L^2(R^n)$,

$$(1.2)\qquad \begin{aligned} D(H_0) &= \{f : (\partial/\partial x)^\alpha f \in L^2(R^n) \text{ for } |\alpha| \leqq 2\}, \\ H_0 f &= -\Delta f \quad \text{for } f \in D(H_0). \end{aligned}$$

The Fourier transform

$$(1.3)\qquad \hat{f}(\xi) = \text{l.i.m.}\,(2\pi)^{-n/2} \int_{R^n} f(x)\, e^{-ix\cdot\xi} dx \quad (\xi \in R^n)$$

is a unitary map

$$L^2(R^n) \ni f \rightarrow \hat{f} \in L^2(R^n)$$

which "diagonalizes" H_0, i.e., which transforms H_0 into multiplication by $|\xi|^2$,

$$(1.4)\qquad (H_0 f)\hat{}\,(\xi) = |\xi|^2 \hat{f}(\xi).$$

The plane wave $e^{ix\cdot\xi}$ is an eigenfunction of the differential operator $-\Delta$,

$$-\Delta e^{ix\cdot\xi} = |\xi|^2 e^{ix\cdot\xi},$$

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