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SCATTERING THEORY[1]

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Introduction. In this series of lectures we shall develop a theory of scattering for first order systems:

$$(1) \qquad u_t = Gu = \sum_{j=1}^{k} A^j \partial_j u + Bu, \qquad \partial_j u = \frac{\partial u}{\partial x_j}, \qquad u(x,0) = f(x)$$

over R^k. Here u is an n-component vector-valued function, $A^j(x)$ and $B(x)$ are $n \times n$ matrix-valued functions depending smoothly on x but independent of t. *We impose the following conditions*:

(1) *The L_2-energy is conserved.* This means that the energy at time t, namely

$$E[u(t)] \equiv \int_{R^k} |u(t)|^2 dx$$

is constant in time. Hence with respect to the energy norm G must be skew-symmetric and this in turn requires that the A^j be Hermitian symmetric and that

$$(2) \qquad B(x) + B^*(x) = \sum_{j=1}^{k} \partial_j A^j(x).$$

In fact

$$\begin{aligned} \frac{d}{dt} E[u] &= \int (\partial_t u \cdot u + u \cdot \partial_t u) dx \\ &= \int \left(\sum_{j=1}^{k} A^j \partial_j u \cdot u + u \cdot A^j \partial_j u + Bu \cdot u + u \cdot Bu \right) dx \\ &= \int \sum_{j=1}^{k} \partial_j (A^j u \cdot u) dx + \int \left(B + B^* - \sum_{j=1}^{k} \partial_j A^j \right) u \cdot u dx. \end{aligned}$$

Integrating over all space the first term in the right vanishes; in order that $dE[u]/dt$ vanish for all smooth data u we see that the relation (2) must hold. Thus if we work in the Hilbert space H of square integrable functions $[L_2(R^k)]^n$, then we can expect that the solution operator

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