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THE ABSTRACT THEORY OF SCATTERING

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1. **Introduction.** This paper deals with the construction and properties of the wave operators $W_{\pm}(H_2, H_1)$ and the scattering operator S associated with two selfadjoint operators H_1 and H_2 in a Hilbert space $\mathcal{H}$. We shall also consider the wave operators $W_{\pm}(U_2, U_1)$ for unitary operators U_1, U_2. More generally, we shall construct wave operators for two spectral measures E_1, E_2 defined on a certain measure space.

There are two main approaches to these problems, called the stationary method and the time-dependent method. The time-dependent method is more convenient for the introduction of the wave and scattering operators. However the stationary method gives more detailed results with fewer assumptions. This paper begins with a summary of the time-dependent approach. The main part of the paper presents an exposition of the stationary method.

2. **A summary of the time-dependent theory of scattering.** Let H_1 and H_2 be selfadjoint operators and consider the associated unitary groups $e^{-itH_1}, e^{-itH_2}, -\infty < t < \infty$. The limits

$$W_{\pm} = \operatorname*{s-lim}_{t \to \pm\infty} e^{itH_2} e^{-itH_1} \tag{2.1}$$

are called the wave operators. Of course such limits will exist only under strong restrictions.

A specific situation, which is typical for applications and to which reference is made frequently below, is the following:

$$\mathcal{H} = L^2(R^3), \quad H_1 = -\Delta, \quad H_2 = -\Delta + V, \quad V = q(x) ,$$

where q is a real-valued measurable function. H_1 is selfadjoint under the standard interpretation of Δ [12, p. 299] and H_2 is selfadjoint under rather mild conditions on q (it suffices if $q \in L^2(R^3) + L^\infty(R^3)$ (vector sum)) [12, p. 302]. These operators correspond to quantum mechanical Hamiltonians for a free particle and a particle moving in the potential field $q(x)$ respectively. In this case $W_{\pm}$ exist if $q(x)$ is sufficiently small for large $|x|$ (precise conditions are given below).

(2.1) implies that

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