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A SPECTRAL COLLOCATION METHOD FOR EIGENVALUE PROBLEMS OF COMPACT INTEGRAL OPERATORS

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ABSTRACT. We propose and analyze a new spectral collocation method to solve eigenvalue problems of compact integral operators, particularly, piecewise smooth operator kernels and weakly singular operator kernels of the form $1/|t-s|^{\mu}$, $0 < \mu < 1$. We prove that the convergence rate of eigenvalue approximation depends upon the smoothness of the corresponding eigenfunctions for piecewise smooth kernels. On the other hand, we can numerically obtain a higher rate of convergence for the above weakly singular kernel for some μ's even if the eigenfunction is not smooth. Numerical experiments confirm our theoretical results.

1. Introduction. We consider numerical approximation of the eigenvalue problem for a compact integral operator T on a Banach space. Recent years have witnessed a revitalization of this field, and various methods are applied to solve the problem. The Galerkin, Petrov-Galerkin, collocation, Nyström and degenerate kernel methods have been extensively studied for the approximation of eigenvalues and eigenvectors of integral operators. The results are well documented in the literature. Here, we mention a few related to our current work. As early as 1967, Atkinson proved a general theorem showing the convergence of numerical eigenvalues and eigenvectors to those of compact integral operators [**2**]. In 1975, he further obtained a convergence rate for the approximation [**3**], based upon which Osborn established a general spectral approximation theory for compact operators, when a sequence of $\{T_n\}$ approximates T in a collectively compact manner. The analysis of [**3, 17**] covers many methods and provides a basis for the convergence analysis of our method. In [**13**], Dellwo and Friedman proposed

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