

DIFFERENTIABILITY OF SOLUTIONS OF ABSTRACT NEUTRAL INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper we study the differentiability of solutions of abstract neutral integro-differential equations with infinite delay. We consider specially the cases when the underlying space is reflexive or at least has the Radon-Nikodym property.

1. Preliminaries. In this work we are concerned with regularity properties of solutions of abstract neutral integro-differential equations with infinite delay.

Let X be a Banach space endowed with a norm $\|\cdot\|$. In this paper we study the existence of classical solutions for the class of abstract neutral integro-differential equations described in the form

$$(1.1) \quad \frac{d}{dt} \left(x(t) + \int_0^t N(t-s)x(s) ds \right) \\ = Ax(t) + \int_0^t B(t-s)x(s) ds + f(t, x_t),$$

for $t \in I = [0, a]$, with initial condition

$$(1.2) \quad x_0 = \varphi \in \mathcal{B}$$

In this description $x(t) \in X$ and the history $x_t : (-\infty, 0] \rightarrow X$, given by $x_t(\theta) = x(t + \theta)$ for $\theta \leq 0$, belongs to some abstract phase space

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