

# MULTILEVEL AUGMENTATION METHODS FOR NONLINEAR BOUNDARY INTEGRAL EQUATIONS II: ACCELERATED QUADRATURES AND NEWTON ITERATIONS

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**ABSTRACT.** A fast multilevel augmentation method (MAM) was proposed recently by the same authors for solving a class of nonlinear boundary integral equations. In this paper, we develop accelerated quadrature formulas for computing the integrals involved in the MAM and approximate iteration for solving the resulting nonlinear system. Specifically, we employ a product integration scheme for computing the singular integrals which appear in the matrices involved in the MAM and introduce an approximation technique in the Newton iteration for solving the resulting nonlinear systems to avoid repeated computation in generating their Jacobian matrices. The use of these two techniques results in a modified MAM which speeds up its computation. We show that the modified MAM preserves the optimal convergence order of the original one while reducing computational costs. Numerical results are presented to demonstrate the approximation accuracy and computational efficiency of the proposed modified MAM, with a comparison to those of the original one and a known algorithm of Atkinson and Chandler.

**1. Introduction.** Boundary value problems of the Laplace equation are commonly used mathematical models for many important applications, such as acoustics, elasticity, electromagnetics, fluid dynamics (see, for example, [20, 23, 24, 27] and the references cited therein).

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