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MULTILEVEL AUGMENTATION METHODS WITH MATRIX COMPRESSION FOR SOLVING REFORMULATED HAMMERSTEIN EQUATIONS

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ABSTRACT. In this paper we supplement matrix truncation strategies with the multilevel augmentation methods for solving the reformulated Hammerstein equations. The resulting numerical solutions have nearly optimal convergence order with linear order computational complexity up to a logarithmic factor with respect to the dimension of the discretization subspace. Numerical experiments on one and two dimensional equations illustrate that our algorithm gains remarkably high efficiency without losing accuracy.

1. Introduction. The Hammerstein equation is an important kind of nonlinear integral equation. It serves as a mathematical model for many applications, such as astrophysics, fluid dynamics, cell kinetics, mathematical economies and so on. Moreover, it is used as a model equation to test numerical methods for solving nonlinear integral equations. There are various numerical methods for solving the Hammerstein equation, which include projection methods, Nyström methods, homotopy analysis, domain decomposition, etc. In [**1**], several popular numerical schemes are reviewed. For more recent developments on the numerical methods, see [**4, 5, 18–20, 22, 28, 29**]. All of these methods involve evaluating the Jacobian matrix. Thanks to the

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