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BLOW-UP BEHAVIOR OF HAMMERSTEIN-TYPE VOLTERRA INTEGRAL EQUATIONS

H. BRUNNER AND Z.W. YANG

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ABSTRACT. In this paper, we consider the blow-up behavior of Hammerstein-type Volterra integral equations. Based on several fundamental assumptions, some necessary and sufficient conditions under which the solution blows up in finite time are given. Some examples illustrate that there may always exist a global solution for a power-law function and that the blow-up behavior only depends upon the value of the kernel in a neighborhood of zero. As an application, we give some results on the blow-up behavior of Volterra integro-differential equations of Hammerstein-type.

1. Introduction. In this paper, we investigate the blow-up behaviors of solutions of Hammerstein-type Volterra equations

$$u(t) = \phi(t) + \int_0^t k(t-s)G(s,u(s))\,\mathrm{d}s, \tag{1.1}$$

where $\phi : [0,\infty) \to [0,\infty)$ and $G : [0,\infty) \times \mathbf{R} \to [0,\infty)$ are continuous functions, the kernel $k : (0,\infty) \to [0,\infty)$ is a locally integrable function and u is an unknown (continuous) solution. We note that results on local existence and uniqueness of solutions for (1.1) may be found in [**16**].

In recent years, the literature includes extensive blow-up results of ordinary differential equations, partial differential equations and

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