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SUPERCONVERGENT NYSTRÖM AND DEGENERATE KERNEL METHODS FOR INTEGRAL EQUATIONS OF THE SECOND KIND

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ABSTRACT. We propose, in this paper, new methods for approximating the solution of a second kind integral equation with a smooth kernel or kernel having a discontinuity along the diagonal. By using an interpolatory projection at Gaussian points onto the space of (discontinuous) piecewise polynomials of degree $\leq r-1$, we prove that the proposed methods exhibit convergence orders $3r$ and $4r$ for the iterated version. In comparison with Kulkarni's of the same convergence order, we show that our methods are faster and simpler to implement. The theoretical results obtained are illustrated by some numerical examples.

1. Introduction. Let us consider the linear integral equation of the second kind

$$u - \mathcal{K}u = f, \tag{1}$$

where $\mathcal{K}$ is the compact linear operator defined on the space $\mathcal{C}[0,1]$ by

$$\mathcal{K}u(s) = \int_0^1 k(s,t)u(t)\,dt, \quad s \in [0,1]$$

with $k(\cdot,\cdot) \in \mathcal{C}([0,1]\times[0,1])$ and $f \in \mathcal{C}[0,1]$.

Assume that the homogenous integral equation $u - \mathcal{K}u = 0$ has, in $C([0,1])$, only the trivial solution; then the operator $(\mathcal{I}-\mathcal{K})$ is invertible and

$$(\mathcal{I}-\mathcal{K})^{-1} = \sum_{n=0}^{\infty} \mathcal{K}^n.$$

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