JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 24, Number 3, Fall 2012

TRACTABILITY OF THE FREDHOLM PROBLEM OF THE SECOND KIND

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Communicated by Ian Sloan

ABSTRACT. We study the tractability of computing ε-approximations of the Fredholm problem of the second kind: given $f \in F_d$ and $q \in Q_{2d}$, find $u \in L_2(I^d)$ satisfying

$$u(x) - \int_{I^d} q(x,y)u(y)\,dy = f(x) \quad \text{for all } x \in I^d = [0,1]^d.$$

Here, F_d and Q_{2d} are spaces of d-variate right hand functions and $2d$-variate kernels that are continuously embedded in $L_2(I^d)$ and $L_2(I^{2d})$, respectively. We consider the worst case setting, measuring the approximation error for the solution u in the $L_2(I^d)$-sense. We say that a problem is tractable if the minimal number of information operations of f and q needed to obtain an ε-approximation is sub-exponential in ε^{-1} and d. One information operation corresponds to the evaluation of one linear functional or one function value. The lack of sub-exponential behavior may be defined in various ways, and so we have various kinds of tractability. In particular, the problem is strongly polynomially tractable if the minimal number of information operations is bounded by a polynomial in ε^{-1} for all d.

We show that tractability (of any kind whatsoever) for the Fredholm problem is equivalent to tractability of the L_2-approximation problems over the spaces of right-hand sides and kernel functions. So (for example) if both of these approximation problems are strongly polynomially tractable, so is the Fredholm problem. In general, the upper bound provided by this proof is essentially non-constructive, since it involves an interpolatory algorithm that exactly solves the Fredholm problem (albeit for finite-rank approximations of f and q). However, if linear functionals are permissible and F_d and Q_{2d} are weighted tensor product spaces, we are able to

This research was supported in part by the National Science Foundation.
Received by the editors on September 21, 2010, and in revised form on January 25, 2011.
DOI:10.1216/JIE-2012-24-3-413