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COMPACTNESS OF LINEAR INTEGRAL OPERATORS IN IDEAL SPACES OF VECTOR FUNCTIONS

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ABSTRACT. Estimates for the measure of noncompactness of linear integral operators of vector functions in ideal spaces are obtained. When the kernel function is compact, no additional uniformity or measurability hypotheses are needed; however, noncompact nonmeasurable kernel functions are also treated.

1. Introduction. Throughout this paper, let T and S be σ-finite measure spaces. Under some continuity or growth assumptions for f, it is well known that

$$Ax(t) := \int_S f(t,s,x(s))\,ds \quad (t \in T) \tag{1}$$

is continuous and compact in C (if T and S are compact subsets of $\mathbf{R}^n$) or in L_p or, more generally, in ideal spaces, respectively, see e.g., [**6**, Part I, Theorems 3.1, 3.2], [**7**, Sections 5, 19] or [**18**].

It is natural to conjecture that, in the case of vector functions x and f, i.e., if $f: T \times S \times U \to V$ with Banach spaces U and V, one obtains similar results. More precisely, one might conjecture that if $f(t,s,\cdot)$ is a compact operator for almost all $(t,s) \in T \times S$ and if f is a Carathéodory function (i.e., $u \mapsto f(t,s,u)$ is continuous for almost all (t,s) and $(t,s) \mapsto f(t,s,u)$ is (strongly) measurable for each $u \in U$) then under natural additional hypotheses the corresponding

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