DISCRETE SUPERCONVERGENCE OF COLLOCATION SOLUTIONS FOR FIRST-KIND VOLTERRA INTEGRAL EQUATIONS

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ABSTRACT. It is known that collocation solutions for first-kind Volterra integral equations based on (discontinuous or continuous) piecewise polynomials cannot exhibit local superconvergence at the points of a uniform mesh. In this paper we present a complete analysis of local superconvergence of such collocation solutions for first-kind Volterra integral equations at non-mesh points. In particular, we discuss (i) the existence of superconvergence points for prescribed collocation points; (ii) the existence of collocation points for prescribed superconvergence points. Numerous examples illustrate the theory.

1. Collocation methods. The convergence properties of collocation and Galerkin solutions in spaces of (discontinuous or continuous) piecewise polynomials for first-kind Volterra integral equations

$$(1.1) \qquad \int_0^t K(t,s)u(s)\,ds = f(t), \quad t \in I := [0,T],$$

with bounded (smooth) kernels $K(t,s)$, are now well understood; see [**2–6, 9–12**] and [**7, 8**], respectively. In particular, it was shown in