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EXISTENCE OF THREE SOLUTIONS TO IMPULSIVE DIFFERENTIAL EQUATIONS

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ABSTRACT. We deal with Dirichlet boundary value problems for impulsive differential equations depending on a parameter λ. Under some assumptions, the existence of at least three solutions is obtained by using a critical point theorem.

1. Introduction. In this paper, we are concerned with the existence of three solutions for the following Dirichlet boundary value problems

$$\begin{aligned} -u''(t) &= \lambda f(u(t)), \quad t \neq t_j,\ t \in [0,1], \\ \Delta u'(t_j) &= I_j(u(t_j)), \quad j = 1, 2, \dots, p, \\ u(0) &= u(1) = 0 \end{aligned} \tag{1.1}$$

where $0 = t_0 < t_1 < \cdots < t_p < t_{p+1} = 1$, $f \in C(R, R)$, $I_j \in C(R, R)$, $j = 1, 2 \dots, p$, $\Delta u'(t_j) = u'(t_j^+) - u'(t_j^-)$, $u'(t_j^+)$ and $u'(t_j^-)$ denote the right and the left limits, respectively, and $\lambda \in [0, +\infty)$ is a real parameter.

In recent years, a great deal of work has been done in the study of the existence of multiple solutions for impulsive boundary value problems; we refer the reader to [**1, 2, 4, 5**]. These classical tools in literature are fixed-point theorems in cones. It is well known that the critical point theorem is an important tool in dealing with problems for differential equations. We also note that, in the last few years, some researchers have used variational methods to study the existence of solutions for impulsive differential equations boundary value problems

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