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FAST SINGULARITY PRESERVING METHODS FOR INTEGRAL EQUATIONS WITH NON-SMOOTH SOLUTIONS

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ABSTRACT. Fast singularity preserving multiscale Galerkin methods are developed in this paper for solving weakly singular Fredholm integral equations of the second kind with non-smooth solutions. A truncation strategy for the coefficient matrix obtained by using singularity preserving multiscale Galerkin methods is proposed. The multilevel augmentation method is developed for solving the discrete system with the truncated matrix. We prove that the methods preserve the singularities of the solutions and possess optimal order of convergence and linear computational complexity (up to a logarithmic factor). Finally, numerical experiments are presented to confirm theoretical results and demonstrate the efficiency and accuracy of the methods.

1. Introduction. Fast wavelet and multiscale methods for numerical solutions of weakly singular integral equations have attracted much attention recently. The methods are based on the fact that the representation of integral operators by appropriate wavelet and multiscale bases produces numerically sparse matrices. Matrix truncation (compression) techniques are then designed, which lead to fast algorithms for solving the integral equations (see, for example, [**1, 6, 7, 14, 15, 22, 23**] and the references cited therein). Moreover, multilevel augmentation methods are proposed as fast solvers for solving the discrete linear

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