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SYSTEMS OF SINGULARLY PERTURBED FRACTIONAL INTEGRAL EQUATIONS

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ABSTRACT. The solution of a singularly perturbed type of a system of fractional integral (differential) equations is studied in this paper. The formal asymptotic solution is derived and proved to be asymptotically correct. Basic matrix algebra is used to prove the asymptotic decay in the inner layer solution.

1. Introduction. Consider the singularly perturbed system

$$\varepsilon\mathbf{u}(t) = \mathbf{g}(t) + {}_0J_t^{\alpha} A(t)\mathbf{u}(t), \quad 0 \le t \le T,\ 0 < \alpha < 1,\ \mathbf{g}(0) = \mathbf{0}. \tag{1.1}$$

The vector valued function $\mathbf{g}(t)$ is continuous for $0 \le t \le T$; the matrix valued function $A(t)$ is also continuous on $0 \le t \le T$ for $T > 0$. The positive parameter ε is considered to be very small, nearly zero.

The operator ${}_\varsigma J_t^{\gamma}$, and later ${}_\varsigma D_t^{\gamma}$ are defined using Riemann-Liouville definition. That, for a continuous function ϕ and for $\varsigma < \gamma < 1$,

(1.2a)
$${}_\varsigma J_t^{\gamma}\phi(t) := \frac{1}{\Gamma(\gamma)} \int_\varsigma^t (t-s)^{\gamma-1}\phi(s)\, ds, \quad t \ge \varsigma,$$

(1.2b)
$${}_\varsigma D_t^{\gamma}\phi(t) := \frac{1}{\Gamma(1-\gamma)} \frac{d}{dt} \int_\varsigma^t (t-s)^{-\gamma}\phi(s)\, ds, \quad t > \varsigma.$$

Mathematical modeling of real life processes using differential and integral equations, has recently been in favor of fractional order models

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