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NONLOCAL INITIAL BOUNDARY VALUE PROBLEM FOR A FRACTIONAL INTEGRODIFFERENTIAL EQUATION IN A BANACH SPACE

A. ANGURAJ AND P. KARTHIKEYAN

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ABSTRACT. In this paper, we study the existence and uniqueness of solutions for fractional integrodifferential equations with nonlocal initial condition in a Banach space. The results are established by the application of the contraction mapping principle and the Krasnoselkii fixed point theorem. An application is also given.

1. Introduction. In this paper, we consider an initial boundary value problem (IBVP for short) for a fractional integrodifferential equation with a nonlocal initial condition, of the form

$$\begin{cases} {}^cD^q x(t) = \int_0^t k(t,s,x(s))\,ds \quad t \in I = [0,1], \\ x(0) = \int_0^1 g(s)x(s)\,ds, \end{cases} \tag{1.1}$$

where ${}^cD^q$ is the standard Caputo fractional derivative of order $0 < q < 1$, and $x : I \to E$ for a Banach space, E. We assume that $g \in L^1([0,1], R_+)$ with $g(t) \in [0,1)$, and k is a given E-valued function satisfying some conditions that will be specified later.

Fractional differential equations have gained considerable importance due to their application in various sciences, such as physics, mechanics, chemistry, engineering, etc. In fact, fractional differential equations are considered as providing alternative models to nonlinear differential equations [**4**]. For more details on the geometric and physical interpretation of fractional derivatives of the Caputo type, see [**5**].

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