

ON SOME NEW GRONWALL–BELLMAN–OU-IANG
TYPE INTEGRAL INEQUALITIES
TO STUDY CERTAIN EPIDEMIC MODELS

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ABSTRACT. We present some new nonlinear Gronwall–Bellman–Ou-Iang type integral inequalities related to Pachpatte’s inequality. [11]. These inequalities generalize former results and can be used as handy tools to study the qualitative behavior as well as of certain quantitative properties of solutions of certain epidemic models and of certain differential equations.

1. Introduction. It is well known that the integral inequalities involving functions of one, and more than one, independent variable which provide explicit bounds on unknown functions, have proved to be very useful and important devices in the study of many qualitative behaviors as well as quantitative properties of solutions of differential and integral equations. In recent years, these inequalities have been greatly enriched by the recognition of their potential and intrinsic worth in many applications of the applied sciences. In the past few years, a number of integral inequalities have been established by many scholars, which were motivated by certain applications. For example, we refer the reader to references [1, 2, 4–7, 12] and some of the references cited therein.

In a paper published in 1981, [9] studied the qualitative behavior of solutions of the equation

$$(1.1) \quad u(t) = k \left[p(t) - \int_0^t A(t-s)u(s) ds \right] \left[q(t) + \int_0^t a(t-s)u(s) ds \right].$$

Keywords and phrases. Gronwall–Bellman’s inequality, Ou-Iang’s inequality, epidemic models, qualitative behavior and quantitative properties for solutions of differential equations.

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