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A DUAL INTEGRAL EQUATION METHOD FOR CAPILLARY-GRAVITY WAVE SCATTERING

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ABSTRACT. A mixed boundary value problem for Laplace's equation involving a higher order boundary condition, associated with scattering (radiation) of capillary-gravity waves in deep water by submerged as well as surface piercing vertical barrier (wave-maker) is considered for its complete solution. Utilizing recently developed mode-coupling relations for eigenfunctions in the expansion formula of the potential function, the boundary value problem has been reduced to solving dual integral equations with kernels comprised of trigonometric functions. A fully analytical solution is derived by the aid of a weakly singular integral equation whose solution has bounded behavior at the end points. The reflection and transmission coefficients, for an incident wave, have been obtained analytically in terms of modified Bessel functions. Numerical results are computed and presented graphically for a surface tension parameter, plotted against a non-dimensional wave parameter. The present method of solution is essentially an extension of the reduction method originally described by Williams [**20**].

1. Introduction. Mixed boundary value problems occurring in the theory of linear water waves involving vertical barriers have been of interest to many research workers. A number of methods of solution are explained for different barrier topographies by Ursell [**18**], Williams [**20**], Evans [**7**] and others. One method of dealing with such boundary value problems is to reduce them to solving dual integral equations. These equations are often encountered in different branches of mathematical physics, and they generally arise while solving a boundary value problem with mixed boundary conditions (see Sneddon [**16**]). Chakrabarti et al. ([**1, 4**]) studied water wave scattering by

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