

ASYMPTOTIC EXPANSIONS FOR
APPROXIMATE SOLUTIONS OF
FREDHOLM INTEGRAL EQUATIONS WITH
GREEN'S FUNCTION TYPE KERNELS

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ABSTRACT. Asymptotic expansions at the node points for approximate solutions of the second kind Fredholm integral equation with a kernel of Green's type function are obtained in the Nyström method based on the composite midpoint, the composite Simpson and the composite modified Simpson rules. Similar expansions are also obtained for the iterated collocation method associated with piecewise constant, piecewise linear and piecewise quadratic functions. Richardson extrapolation is used to obtain approximate solutions with higher order of convergence at the node/partition points. Numerical examples are given to illustrate various results.

1. Introduction. Asymptotic expansions of approximate solutions of second kind Fredholm integral equations with a smooth kernel have been extensively studied in the research literature. Some of the important methods for finding an approximate solution are Nyström methods defined by replacing the integral in the integral operator by a convergent quadrature formula and projection related methods such as the classical Galerkin method and its variants. Asymptotic expansions in the case of the Nyström method associated with various composite quadrature rules have been obtained by Baker [4] and McLean [11]. They also consider the iterated Galerkin and the iterated collocation methods associated with projections onto a piecewise polynomial space with respect to a uniform partition. These expansions are based on

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