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VOLTERRA INTEGRAL EQUATIONS AND FRACTIONAL CALCULUS: DO NEIGHBORING SOLUTIONS INTERSECT?

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ABSTRACT. In this paper we consider the solutions to two neighboring Hammerstein-type Volterra integral equations of the form

$$y(t) = \sigma + \int_0^t p(t,s) f(s, y(s))\, ds; \quad \sigma = y_0, z_0.$$

We give a theorem that guarantees that the solutions never intersect if $y_0 \neq z_0$, and we discuss several consequences of the main theorem that concern initial and boundary value problems for fractional calculus. Finally, we give an example that illustrates how one may calculate the history of the solution to a boundary value fractional differential equation.

1. Introduction. In this paper we consider the question of whether or not the solutions to two Volterra integral equations which have the same kernel but different forcing terms may intersect at some future time. Our discussions are motivated by the desire to set out a fairly general framework in which existing results about the intersection of solutions to ordinary differential equations can be extended to related problems such as solutions to equations of fractional order.

The Volterra second kind integral equations that we shall consider take the Hammerstein form

$$y(t) = \sigma + \int_0^t p(t,s) f(s, y(s))\, ds \tag{1}$$

with some constant $\sigma \in \mathbf{R}$ where f is assumed to be continuous whereas p may be singular. Equations of this type have been analyzed by many

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