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POSITIVE SOLUTIONS FOR A SYSTEM OF NONLINEAR HAMMERSTEIN INTEGRAL EQUATIONS AND APPLICATIONS

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ABSTRACT. This paper deals with the existence and multiplicity of positive solutions for the system of nonlinear Hammerstein integral equations

$$\begin{cases} u(x) = \int_0^1 k(x,y) f_1(y, u(y), v(y), w(y))\,\mathrm{d}y, \\ v(x) = \int_0^1 k(x,y) f_2(y, u(y), v(y), w(y))\,\mathrm{d}y, \\ w(x) = \int_0^1 k(x,y) f_3(y, u(y), v(y), w(y))\,\mathrm{d}y. \end{cases}$$

We use concave functions to characterize growing and interacting behaviors of our nonlinearities so that f_1, f_2, f_3 cover three cases: the first with all superlinear, the second with all sublinear, and the last with two superlinear and the other sublinear. Based on a priori estimates achieved by using Jensen's integral inequality, we use fixed point index theory to establish our main results. As an application, we use our main results to establish the existence and multiplicity of positive solutions for a system of nth order boundary value problems for nonlinear ordinary differential equations.

1. Introduction. In this paper we study the existence and multiplicity of positive solutions for the following system of nonlinear Hammerstein integral equations

$$\begin{cases} u(x) = \int_0^1 k(x,y) f_1(y, u(y), v(y), w(y))\,\mathrm{d}y, \\ v(x) = \int_0^1 k(x,y) f_2(y, u(y), v(y), w(y))\,\mathrm{d}y, \\ w(x) = \int_0^1 k(x,y) f_3(y, u(y), v(y), w(y))\,\mathrm{d}y, \end{cases} \tag{1.1}$$

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