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APPLICATIONS OF GENERALIZED CONVOLUTIONS ASSOCIATED WITH THE FOURIER AND HARTLEY TRANSFORMS

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ABSTRACT. In this paper we present new generalized convolutions with weight-function associated with the Fourier and Hartley transforms, and consider applications. Namely, using the generalized convolutions, we construct normed rings on the space $L^1(\mathbf{R}^d)$, provide the sufficient and necessary condition for the solvability of a class of integral equations of convolution type, and receive the explicit solutions of those equations.

1. Introduction. The theory of convolutions has been studied for a long time and applied to many fields of mathematics. In recent years, many convolutions, generalized convolutions, and poly-convolutions of the well-known transforms as the Fourier, Hankel, Mellin, Laplace, and the applications of those transforms have been published (see [**2–7, 11–13, 18, 27, 33**]). Loosely speaking, each one of generalized convolutions is a new integral transform which may be an object of study; for instance, the Hilbert transform can be thought of as a convolution of $f(t)$ with the function $g(t) = 1/(\pi t)$, and the Weierstrass transform is exactly the convolution of that function with the Gaussian function $e^{-t^2/4}$. As Kakichev stated in his paper [**23**], many generalized convolutions of known transforms have not been found.

The main aims of this paper are to present generalized convolutions with the Gaussian weight-function $\gamma(x) = e^{-|x|^2/2}$ associated with the

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