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CONVERGENCE OF ADAPTIVE BOUNDARY ELEMENT METHODS

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ABSTRACT. In many applications, adaptive mesh-refinement is observed to be an efficient tool for the numerical solution of partial differential equations and integral equations. Convergence of adaptive schemes to the correct solution, however, is so far only understood for certain kind of differential equations. In general, it cannot be excluded that the adaptive algorithm computes a *convergent sequence* of discrete approximations with a limit which is *not* the correct solution. This work proposes a feedback loop which guarantees the convergence of the computed discrete approximations to the correct solution. Although stated for Symm's integral equation of the first kind, the main part of this work is written for a general audience in the context of weak forms as Riesz representations in Hilbert spaces. Numerical examples illustrate the adaptive strategies.

1. Symm's integral equation, introduction, and outline.

1.1. Symm's integral equation of the first kind with the single-layer potential operator. Let Ω be a bounded domain in $\mathbf{R}^d$, $d = 2, 3$, with Lipschitz boundary $\partial\Omega$, and let $\Gamma \subset \partial\Omega$ be an open or closed surface. Suppose we are given the right-hand side f and an approximation ϕ_h for the unknown exact solution ϕ of Symm's integral equation of the first kind

$$V\phi = f \quad \text{in } H^{1/2}(\Gamma) \tag{1.1}$$

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