JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 23, Number 4, Winter 2011

BOUNDARY BEHAVIOR OF THE LAYER POTENTIALS FOR THE TIME FRACTIONAL DIFFUSION EQUATION IN LIPSCHITZ DOMAINS

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Communicated by William McLean

ABSTRACT. This paper investigates the boundary behavior of layer potentials for the time fractional diffusion equation (TFDE) in Lipschitz domains. The paper is a continuation of [**9**]. Since now the boundary of the spatial domain Ω admits only Lipschitz smoothness, we have to replace the classical technique used in [**9**] with a more delicate harmonic analysis technique.

We prove that certain nontangential maximal functions related to the layer potentials are bounded in $L^p(\Sigma_T)$, which in particular implies the usual jump relations known for the heat equation. Although the results are well known in the case of the heat potential corresponding to the case $\alpha = 1$, the proofs of the same properties seem not to be available in the case $0 < \alpha < 1$.

1. Introduction. We study the boundary behavior of layer potentials for the time fractional diffusion equation

$$\begin{aligned} \partial_t^\alpha \Phi - \Delta\Phi &= 0, \quad \text{in } Q_T = \Omega \times (0,T), \\ \Phi &= g, \quad \text{on } \Sigma_T = \Gamma \times (0,T) \\ \Phi(x,0) &= 0, \;\; x \in \Omega, \end{aligned} \tag{1}$$

where $\Omega \subset \mathbf{R}^n$ is a bounded domain with Lipschitz boundary Γ and

$$\partial_t^\alpha u(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\alpha} u'(\tau)\,\mathrm{d}\tau \tag{2}$$

is the fractional Caputo time derivative of order $0 < \alpha \le 1$. For $\alpha = 1$ the fractional derivative is interpreted as the limit $\lim_{\alpha\uparrow 1} \partial_t^\alpha u(t)$, which coincides with the usual time derivative $\mathrm{d}u(t)/\mathrm{d}t$ [**8**, page 68].

Received by the editors on June 6, 2009, and in revised form on November 1, 2009.

DOI:10.1216/JIE-2011-23-4-541