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SPECTRUM OF A VISCOELASTIC BOUNDARY DAMPING MODEL

W. DESCH, E. FAŠANGOVÁ, J. MILOTA AND G. PROPST

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ABSTRACT. The undamped linear wave equation with convolution boundary conditions is considered. Physically, the boundary condition models the interaction of a viscoelastic boundary material with memory and the incident waves. We consider three geometric configurations: an interval, a disc and a cylinder. The main result is that the rate of exponential decay can be arbitrarily low. This is demonstrated by proving the existence of eigenvalues that are arbitrarily close to the imaginary axis. The assumptions on the convolution kernel a are: it is of positive type and its Laplace transform $\widehat{a}(\lambda)$ tends to zero when $\Im\lambda$ goes to ∞ while $\Re\lambda$ stays bounded.

1. Introduction. Consider a model for the evolution of sound in a compressible fluid with viscoelastic surface (cf. [**5**]):

$$
\begin{aligned}
&p_{tt}(t,x) - \Delta p(t,x) = 0, \quad x \in \Omega, \\
&\frac{\partial p}{\partial n}(t,x) + a \star p_t(t,x) = 0, \quad x \in \partial\Omega,
\end{aligned}
\tag{1}
$$

where $p(t,x) \in \mathbf{R}$ denotes acoustic pressure, $\Omega \subset \mathbf{R}^3$ is a domain with smooth boundary and $n(x)$ the outer normal to $\partial\Omega$ at x. The convolution is $a \star v(t,.) := \int_{-\infty}^{t} a(t-s)v(s,.)\,ds$, a is a given real-valued function on $[0,\infty)$.

One can consider either solutions on the line ($t \in \mathbf{R}$), or solutions on the halfline ($t \in [0,\infty)$) with initial conditions $p(0,.) = p_0(.)$,

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