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# TWO-SCALE FINITE ELEMENT DISCRETIZATIONS FOR INTEGRODIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper we propose and analyze a number of two-scale discretization schemes for integrodifferential equations arising in finance. It is shown theoretically and numerically that the number of degrees of freedom of the two-scale discretization is significantly smaller than that of the standard one-scale finite element approach while at the same time preserving the accuracy of the one-scale discretization. The main idea of these algorithms is to use a coarse grid to approximate the low frequencies and then to use a fine grid to correct the relatively high frequencies. As a result, both the computational time and the storage can be reduced considerably. A combination of wavelet and Lagrangian finite element basis functions is applied to further reduce the complexity arising from the non-locality of the integrodifferential operators.

## 1. Introduction.

In *mathematical finance*, consider a basket of $d \geq 1$ risky assets whose log returns $X_t$ at time $t > 0$ are modeled by a Lévy process $X = \{X_t\}_{t \geq 0}$ with state space $\mathbf{R}^d$. By the fundamental theorem of asset pricing [**17**], the arbitrage free price $u$ of a European contingent claim with payoff function $g(\cdot)$ and maturity $T > 0$ is given by the conditional expectation

$$u(t,x) = \mathbf{E}\left(g(X_T) \ : \ X_t = x\right),$$

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