JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 23, Number 2, Summer 2011

# INTEGRO-DIFFERENTIAL EQUATIONS OF FIRST ORDER WITH AUTOCONVOLUTION INTEGRAL II

LOTHAR VON WOLFERSDORF AND JAAN JANNO

Communicated by Jurgen Appell

ABSTRACT. In the paper two integro-differential equations of first order with autoconvolution integral and singular coefficients are investigated. For these equations the existence of a solitary solution is proved.

**1. Introduction.** Continuing our paper [**5**] (which we cite as part I of this paper in the following) we deal with two equations of the form

$$y'(x) + \frac{\gamma}{x}y(x) = a(x)\int_0^x y(\xi)y(x-\xi)\,d\xi, \quad x \in (0,T) \tag{1.1}$$

with given coefficient $a$ and nonzero numbers $\gamma < 1$, $T \in (0,1)$. As reported in part I of the paper, equations of form (1.1) have applications in Burgers' turbulence theory. For functions $a$ with singularity at $x=0$ equation (1.1) is of type II in [**5**] and from Theorems 5 and 6 of [**5**] (cf. [**5**, 4.3]) we have the existence of a one-parametric family of solutions $y_K$, $K \in \mathbf{R}$, with $x^\gamma y_K \in C[0,T]$ and $y_K(x) \sim Kx^{-\gamma}$ as $x \to 0$ if

$$x^{1-\gamma}a(x) \in L^1(0,T) \quad \text{for } 0<\gamma<1, \qquad xa(x) \in L^1(0,T) \quad \text{for } \gamma<0, \tag{1.2}$$

respectively. (For $K=0$ this is the trivial solution.) If (1.2) is not fulfilled, in general, we only know the trivial solution $y_0(x) \equiv 0$ for equation (1.1).

---

2010 AMS *Mathematics subject classification.* Primary 45J05, Secondary 45G10, 45D05.
*Keywords and phrases.* Integro-differential equations, autoconvolution equations.
The first author, Lothar von Wolfersdorf, passed away on November 30, 2010. This paper is dedicated to his memory.
The work of Jaan Janno was supported by the Estonian Science Foundation, Grant no. 7728.
Received by the editors on July 14, 2008, and in revised form on March 13, 2009.
DOI:10.1216/JIE-2011-23-2-331 