

**EXISTENCE RESULTS FOR NEUTRAL
INTEGRO-DIFFERENTIAL EQUATIONS WITH
UNBOUNDED DELAY**

JOSÉ PAULO C. DOS SANTOS, HERNÁN HENRÍQUEZ
AND EDUARDO HERNÁNDEZ

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ABSTRACT. In this paper we discuss the existence of mild, strict and classical solutions for a class of abstract integro-differential equations in Banach spaces. Some applications to ordinary and partial integro-differential equations are considered.

1. Introduction. Let $(X, \|\cdot\|)$ be a Banach space. In this paper we study the existence of mild, strict and classical solutions for a class of abstract neutral integro-differential equations with infinite delay described in the form

$$(1.1) \quad \begin{aligned} \frac{d}{dt} \left[x(t) + \int_{-\infty}^t N(t-s)x(s) ds \right] \\ = Ax(t) + \int_{-\infty}^t B(t-s)x(s) ds + f(t, x_t), \quad t \in [0, a], \end{aligned}$$

$$(1.2) \quad x_0 = \varphi \in \mathcal{B},$$

where $A, B(t)$, $t \geq 0$, are closed linear operators defined on a common domain $D(A)$ which is dense in X , $N(t)$ ($t \geq 0$) are bounded linear operators on X , the history $x_t : (-\infty, 0] \rightarrow X$, given by $x_t(\theta) = x(t+\theta)$,

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