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A NYSTRÖM METHOD FOR FREDHOLM INTEGRAL EQUATIONS ON THE REAL LINE

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ABSTRACT. The authors introduce a new procedure for the numerical treatment of Fredholm equations of the second kind on the real axis, based on a Nyström method. The convergence of the method is proved and a priori estimates of the error are given. The case of kernels containing a Hilbert transform is also considered.

1. Introduction. This paper concerns the numerical treatment of the class of integral equations defined by

$$f(y) - \mu \int_{\mathbf{R}} \Gamma(x,y) f(x) w(x)\, \mathrm{d}x = g(y), \quad y \in \mathbf{R}, \tag{1.1}$$

where

$$\Gamma(x,y) = k(x,y) + \nu \int_{\mathbf{R}} \frac{\eta(x,t)\sqrt{w(t)}}{t-y}\, \mathrm{d}t, \tag{1.2}$$

μ and ν are real parameters, w is a Freud weight, k, η and g are given continuous functions, f is an unknown function. The integral in (1.2) is understood in the Cauchy principal value sense, i.e., it defines the Hilbert transform of the function $\eta(x,\cdot)\sqrt{w(\cdot)}$.

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