

## A BOUNDARY INTEGRAL EQUATIONS APPROACH FOR MIXED IMPEDANCE PROBLEMS IN ELASTICITY

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**ABSTRACT.** Direct scattering problems for partially coated obstacles in linear elasticity lead to interior and exterior mixed impedance boundary value problems for the equations of steady-state elastic oscillations. We employ the potential method and reduce the mixed impedance problems to equivalent boundary pseudodifferential equations for arbitrary values of the oscillation parameter. We give a detailed analysis of the corresponding pseudodifferential equations which live on a proper submanifold of the boundary of the elastic body and establish uniqueness and existence results for the original mixed impedance problems for arbitrary values of the oscillation parameter; this is crucial in the study of inverse elastic scattering problems for partially coated obstacles. We also investigate regularity properties of solutions near the curves where the boundary conditions change and establish almost best Hölder smoothness results.

**1. Introduction.** In this paper we investigate the three-dimensional mixed impedance interior and exterior boundary value problems (BVPs) for the equations of steady-state elastic oscillations. We consider an elastic body occupying either an interior bounded domain or its complement. We assume that the simply connected boundary of this domain is divided into two parts, a Dirichlet (rigid) one and a Robin (impedance) one. On the Dirichlet part of the boundary, the displacement vector is given, while on the Robin part, a specific combination—physically expressing the proportionality relation between the displacement and the

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