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FAST MULTILEVEL AUGMENTATION METHODS WITH COMPRESSION TECHNIQUE FOR SOLVING ILL-POSED INTEGRAL EQUATIONS

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ABSTRACT. In this paper, multilevel augmentation methods with compression technique are developed for solving ill-posed integral equations. The methods are based on the combination of Lavrentiev regularization and multiscale Galerkin methods, and lead to fast solutions of the discrete equations. We provide *a priori* error analysis for the methods, propose an *a posteriori* regularization parameter choice strategy using compression technique, and establish optimal convergence rates for approximation solutions. Finally, numerical results are presented to illustrate the efficiency of the method.

1. Introduction. Many problems in science and engineering can be formulated as ill-posed Fredholm integral equations of the first kind. These equations are normally treated by regularization methods such as Tikhonov regularization and Lavrentiev regularization (see, for example, [**11, 12, 22**]). When solving the regularization equations by iteration methods or using discrepancy principles to determine the regularization parameters, we have to solve the regularization equations repeatedly, so developing an efficient fast solver for numerically solving such problems is an important and challenging task. This is what we try to do in this paper.

Multilevel methods are popular for the solution of well-posed problems such as Fredholm integral equation of the second kind (see, for example, [**1, 4, 5, 9, 21**] and the references cited therein). These methods have considerable advantages and have been becoming stan-

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